UGC NET Paper 2 - Computer Science and Applications - Code No.:(87)

Unit 1: Discrete Structures and Optimization

1. Mathematical Logic
2. **Sets and Relations**
3. Counting, Mathematical Induction and Discrete Probability
4. Group Theory
5. Graph Theory
6. Boolean Algebra
7. Optimization

2. Sets and Relations

1. Set Operations
2. Representation and Properties of Relations
3. Equivalence Relations
4. Partially Ordering.

Set: A set is the collection of individual elements in the domain D.

Cardinality of a Set:

Number of elements in a set

Cardinality of a set S, denoted by |S|

Eg.: X={1,2,3} and Y={2,3,1} here, |X|=|Y|

i.e., cardinal number of set X = cardinal number of set Y = 3

Representation of a Set		
Example of set	**Roster or Tabular Form**	**Set Builder Notation**
vowels in English	A={a,e,i,o,u}	A={x:x is a vowel in English}
natural numbers	A={1,2,3,4,.....}	A={x:x∈N}
integers	A={.....,−3,−2,−1,0,1,2,3,.....}	A={x:x∈Z}
positive integers	A={0,1,2,3,.....}	A={x:x∈Z^+}

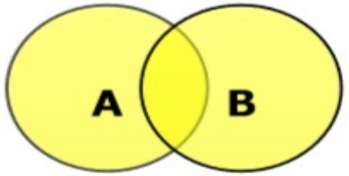
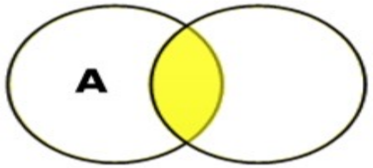
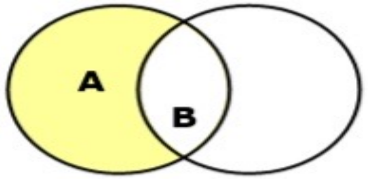
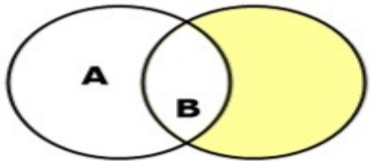
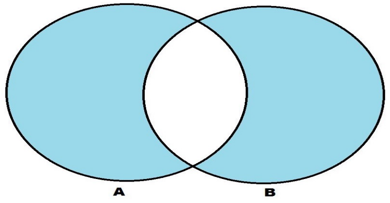
Types of Sets:

1. **Finite**: Eg. X={1,2,3}
2. **Infinite**: Eg: Set of whole numbers i.e: W={0,1,2,3,…}
3. **Empty Set / Null Set / Void Set** : Eg: A=Φ
4. **Singleton Set / Unit Set**: Eg: $S=\{x|x\in N, 7<x<9\} = \{8\}$
5. **Universal Set** U:
   1. Superset of all the sets under consideration.
   2. Eg.: Let U=Universal Set and A be any set then A' is compliment of A given by A'={x:x ∉ A and x ∉ U}
   3. Let U={1,2,3,4,5} and A={1,4} then A'={2,3,5}
6. **subset** X⊆Y:
   1. if every element of X is an element of set Y.
   2. if set A has n elements, then total number of subset of A is 2^n
   3. Eg: A={1,2}, then subset of A are - {}, {1}, {2} and {1,2}
7. **proper subset** X⊂Y:
   1. if every element of X is an element of set Y and |X|<|Y|.
   2. (subset of the set but not equal to)
8. **Power Set**:
   1. If set A has n elements then power set P(A) will have 2^n elements
   2. Notation: $n[P(A)]= 2^n$ Where |A|=n
   3. Eg: If S={a,b,c} then P(S)={Φ,{a},{b},{c},{a,b},{a,c},(b,c},{a,b,c}} where |S|=3 ∴ P(S) has $2^3 = 8$ element

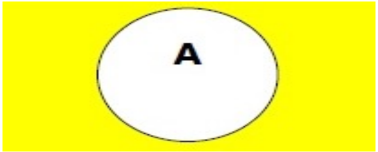
9. Equal Set	10. Equivalent Set
If two sets contain the same elements they are said to be equal.	If the cardinalities of two sets are same, they are called equivalent sets.
Eg: A={1,2,6} and B={6,1,2}	Eg: A={1,2,6} and B={16,17,22} . Since |A| = |B|

11. Overlapping Set	12. Disjoint Set
Two sets that have at least one common element are called overlapping sets.	Two sets A and B are called disjoint sets if they do not have even one element in common.
Properties of Overlapping Set • n(A∪B)=n(A)+n(B)−n(A∩B) • n(A∪B)=n(A−B)+n(B−A)+n(A∩B) • n(A)=n(A−B)+n(A∩B) • n(B)=n(B−A)+n(A∩B)	• Properties of Disjoint Set • n(A∩B)=∅ • n(A∪B)=n(A)+n(B)
Eg: A={1,2,6} and B={6,12,42} common element '6'	Eg: A={1,2,6} and B={7,9,14} not a single common element

Set Operations

1	**Set Union:** $\cup$	$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$	If $A = \{10,11,12,13\}$ and $B = \{13,14,15\}$ then $A \cup B = \{10,11,12,13,14,15\}$	
2	**Set Intersection:** $\cap$	$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$	If $A = \{11,12,13\}$ and $B = \{13,14,15\}$ then $A \cap B = \{13\}$	
3	**Set Difference/ Relative Complement** $A - B$ or $A \setminus B$	$A - B = \{x \mid x \in A \text{ AND } x \notin B\}$	If $A = \{10,11,12,13\}$ and $B = \{13,14,15\}$ then $(A - B) = \{10,11,12\}$ Note: Here, $(A - B) \neq (B - A)$	A-B  B-A 
4	**Symmetric Difference:** $A \Delta B$	$A \Delta B = (A - B) \cup (B - A)$ Note: $A \Delta B$ does not include elements that are in $A \cap B$	If $A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$ $A - B = \{1,2\}$ and $B - A = \{5,6\}$ then, $A \Delta B = \{1,2,5,6\}$	

Set Operations

5	**Complement of a Set**	A′={x|x∉A} A′=(U−A) where U is universal set	If A={x|x ∈ Whole_No. Set} then A′={y|y ∉s Whole_No.Set}	
6	**Cartesian Product / Cross Product**	Eg: If A={a,b} and B={1,2}}, Then Cartesian product of A and B is written as – A×B={(a,1),(a,2),(b,1),(b,2)} Cartesian product of B and A is written as – B×A={(1,a),(1,b),(2,a),(2,b)}		
7	**Partitioning of a Set**	Partition of a set, say *S*, is a collection of *n* disjoint subsets, say P1,P2,…Pn*P*1,*P*2,…*Pn* that satisfies the following three conditions − 1. Pi does not contain the empty set. i.e., [Pi≠{∅} for all 0<i≤n] 2. The union of the subsets must equal the entire original set. i.e., [*P*1∪*P*2∪⋯∪*Pn*=*S*] 3. The intersection of any two distinct sets is empty. i.e., [Pa∩Pb={∅}, for a≠b where n≥a,b≥0] Eg: Let S={a,b,c,d,e,f,g,h} then One probable partitioning is {a},{b,c,d},{e,f,g,h}		

Bell Numbers (denoted by Bn): Bell numbers give the count of the number of ways to partition a set.
Eg: Let S={1,2,3}, n=|S|=3. The alternate partitions are −

1. ∅,{1,2,3}
2. {1},{2,3}
3. {1,2},{3}
4. {1,3},{2}
5. {1},{2},{3}

Hence B_3=5

Relations: Relations may exist between objects of the same set or between objects of two or more sets.

Notation: aRb means (a,b)∈R and b¬Ra means (a,b)∉R

Types of relations:

1. Binary Relation
A binary relation R on 2 sets A and B is a subset of A×B. Eg: Given A={1,2,9} B={1,3,7} Then, A×B = {(1,1),(1,3),(1,7),(2,1),(2,3),(2,7),(9,1),(9,3),(9,7)} If R is relation where (a,b)∈R if and only if a=b then R={(1,1)}. Hence R⊆ AXB
A binary relation R on a single set A is a subset of A×A Eg: Given A={1,2,9, B={1,3,7} Then, A×A={(1,1),(1,2),(1,9),(2,1),(2,2),(2,9),(9,1),(9,2),(9,9)} R is relation where (a,b)∈R if and only if a=b then R={(1,1)}. Hence R⊆ AXB

2. Empty Relation / Void Relation: Relation R = ∅, on set A and B

3. **Full Relation / Universal Relation**: Relation R= A×B, on set A and B

4. **Inverse Relation**: R′={(b,a)|(a,b)∈R} Eg: If R={(1,2),(2,3)} then R′ will be {(2,1),(3,2)}

5. **Identity Relation:** I_A={(1,1),(2,2),(3,3)} Eg: If A={1,2,3}. Then, I_A={(1,1),(2,2),(3,3)}

Reflexive v/s Identity Relation:
Reflexive Relation can include additional pairs beyond the self-related pairs. While identity relation is a specific type of reflexive relation which include self-related pairs only. Eg: If A={1,2,3}. Then, I_A={(1,1),(2,2),(3,3)} is a Identity Relation and also reflexive Relation While R={(1,1),(2,2),(3,3),(1,2),(2,1)} is a reflexive Relation but not an Identity Relation

Properties of Relations

Reflexive Relation	∀ a ∈ A, (a,a) ∈ R or aRa	Let A={1,2,3}. Then R= {(1,1),(2,2),(3,3),(1,2),(2,1)}
Irreflexive relation / Anti-Reflexive Relation	∀ a ∈ A, (a,a) ∉ R or a¬Ra	Let A={1,2,3}. Then R= {(2,3),(3,4)}
Symmetric relation	if (a,b)∈R => (b,a)∈R i.e., xRy => yRx	Let A={1,2,3,4} then R={(1,2),(2,1),(3,4)(4,3)}
Anti-Symmetric relation	if (a,b)∈R and (b,a)∈R => a=b i.e., xRy and yRx => a=b	Let A={1,2,3,4} then R={(1,2),(3,3),(4,3)}
Asymmetric relation	if (a,b)∈R => (b,a)∉R i.e., i.e., xRy => y¬Rx	Let A={1,2,3,4} then R={(1,2),(2,3),(3,4)}
Transitive relation	if (a,b)∈R and (b,c)∈R => (a,c)∈R i.e., xRy and yRz => xRz	Let A={1,2,3} then R={(1,2),(2,3),(1,3)}

Equivalence Relation: A relation R is an Equivalence Relation if and only if it is:

- Reflexive, i.e, $\forall$ a $\in$ A, (a,a) $\in$ R
- Symmetric: if (a,b)∈R => (b,a)∈R
- Transitive, i.e., if (a,b)∈R and (b,c)∈R => (a,c)∈R

Partially Ordering: A relation R is Partially Ordering if and only if it is:

- Reflexive, i.e, $\forall$ a $\in$ A, (a,a) $\in$ R
- Anti-symmetric: if (a,b)∈R and (b,a)∈R => a=b
- Transitive, i.e., if (a,b)∈R and (b,c)∈R => (a,c)∈R

Number of possible relations on 2 sets:

- Let A and B be 2 sets with m and n elements respectively
- Then number of elements of AxB is mn
- Therefore, number of elements in power set of AxB is 2^{mn}
- Thus, AxB has 2^{mn} different subsets.
- Hence No. of possible relations on set A and B are 2^{mn}

Eg: Let A={1,2} and B={1,3,7} Then, calculate number of possible relation from set A to B?

  - |A|=2 , |B|=3 ∴|AxB|=2x3=6
  - Hence number of elements in power set of AxB = number of possible sub-sets of AxB = number of possible relation from set A to B = 2^6 = 64

<u>**Representation of Relations**</u>

1. Relation as a Matrix:

Let P = {a1,a2,a3,.......am}, Q = {b1,b2,b3......bn} and R is a relation from P to Q.

Then R can be represented by m x n matrix M = [Mij], defined as $M_{ij} = \begin{cases} 0 \text{ if } (ai,bj) \notin R \\ 1\ if (ai,bj) \in R \end{cases}$

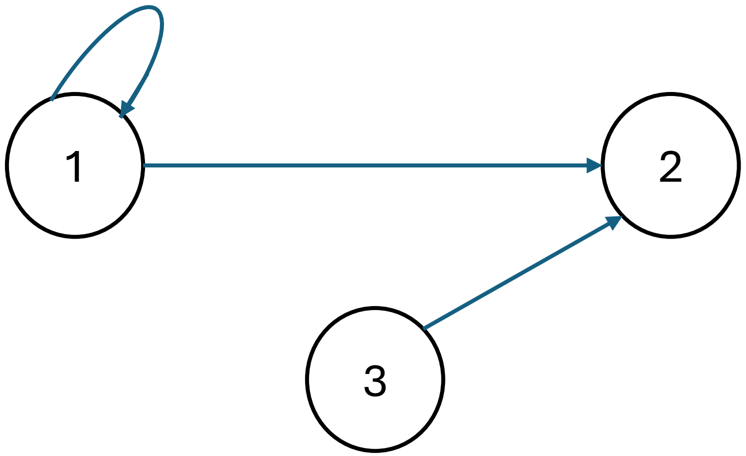
Eg: Let P = {1, 2, 3, 4}, Q = {a, b, c, d} and R = {(1, a), (1, b), (1, c), (2, b), (2, c), (2, d)}.

$M_R =$

P\Q	a	b	c	d
1	1	1	1	0
2	0		1	1
3	0		0	0
4	0	0	0	0

2. Relation as a Directed Graph

Eg: R={(1,1),(1,2),(3,2)} is a relation on set S={1,2,3}



3. Relation as a Table

Eg: Let P = {1, 2, 3, 4}, Q = {x, y, z, k} and R = {(1, x), (1, y), (2, z), (3, z), (4, k)}.

	x	y	z	k
1	x	x		
2			x	
3			x	
4				x

4. Relation as Mapping:

Mapping: represents rule or relationship between sets.

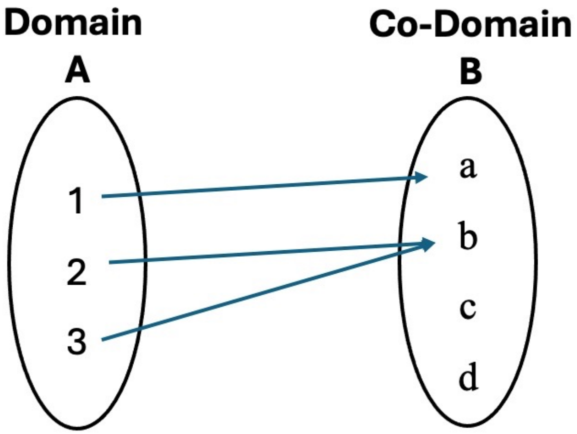
- Domain: set of all input values for which the mapping is defined.
- Codomain: set of all possible output values that can result from the mapping
- Range: set of all output values that are actually produced by the mapping when elements from the domain are used as inputs.

Eg: Consider a mapping M={(1,a),(2,b),(3,b)} from set A={1,2,3} to set B={a,b,c,d}

Domain: A={1,2,3}

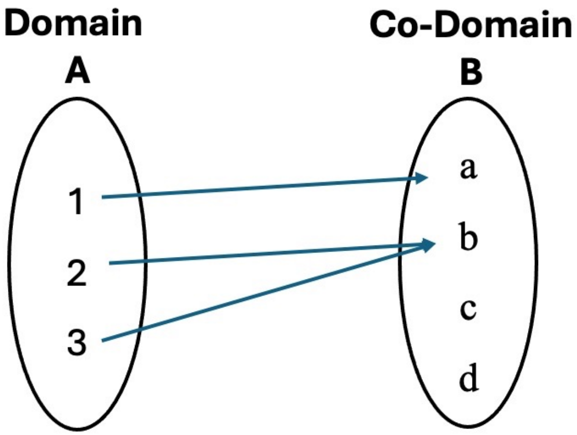
Codomain: B={a,b,c,d}

Range: {a,b}

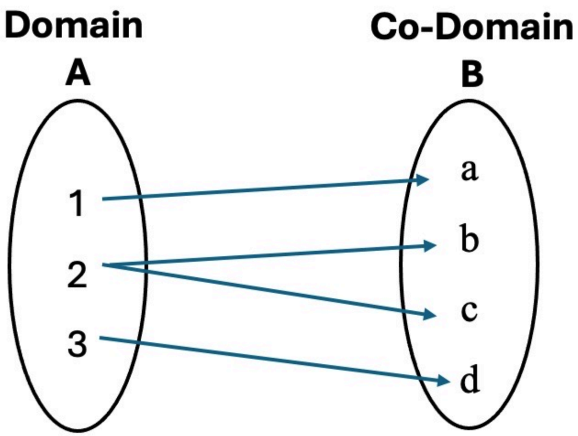


Function:

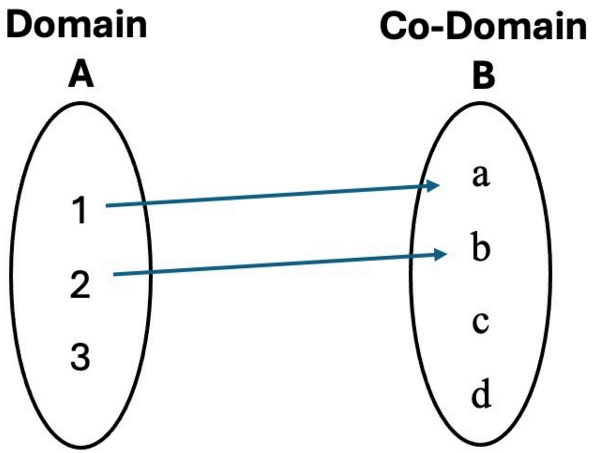
- Function v/s relation/mapping: A function is a unique valued relation/mapping. i.e., every element of A is mapped to only one element of B. However element of B maybe related to more than one element of A.
- Notation: f:A->B , means that f is a mapping that takes all the element of A and match each to a unique element of
- Image/f-image/function-image and Pre-image:
  - Every element of domain has exactly one image in codomain.
  - Elements of co-domin may have one or more pre-images in Domain.
  - f(A) ⊆ B and F ⊆ AxB where, f(A) = Range of f



Mapping
Function



Mapping
Not Function



Not Mapping

Inverse of a Function: f:A→B then inverse of f is g:B→A i.e., f(x)=y⇔g(y)=x
Eg: f:Z→Z,f(x)=x+5 then inverse function g:Z→Z,g(y)=y−5
f(x)=x+5 = > y=x+5 => y-5=x => x=y-5 => g(y)=y-5

Composition of Functions: Two functions f:A→B and g:B→C can be composed to give a composition:

- fog. fog:A->C defined by (fog)(x)=f(g(x)) or
- gof:C->A defined by gof(x)=g(f(x))

Eg: Let f(x)=x+2 and g(x)=2x+1, find (fog)(x) and (gof)(x)	
For (fog)(x) f(x)=x+2 f(g(x))=(g(x)) + 2 f(g(x))=(2x+1) + 2 {given: g(x)=2x+1) f(g(x))=2x+3 (fog)(x)=2x+3 {we know: (fog)(x)=f(g(x)) }	For (gof)(x) g(x)=2x+1 g(f(x))=2(f(x))+ 1 g(f(x))=2(x+2)+ 1 {given: f(x)=x+2 } g(f(x))=2x+4+1 g(f(x))=2x+5 (gof)(x)=2x+5 {we know: (gof)(x)=g(f(x)) }

Note: If f and g are one-to-one then fog and gof are also one-to-one
If f and g are onto then fog and gof are also onto
Composition always holds associative property i.e., (f∘g)∘h=f∘(g∘h)
Composition does not hold commutative property i.e., (f∘g) (g∘f)

Types of mapping:

1. Injective / One-to-one mapping: A function $f:A \rightarrow B$ is injective, if $\forall\ b \in B$, $\exists$ at most one $a \in A$
i.e., f is injective if $a1 \neq a2 \Rightarrow f(a1) \neq f(a2)$ or equivalently f is injective if $f(a1)=f(a2) \Rightarrow a1=a2$

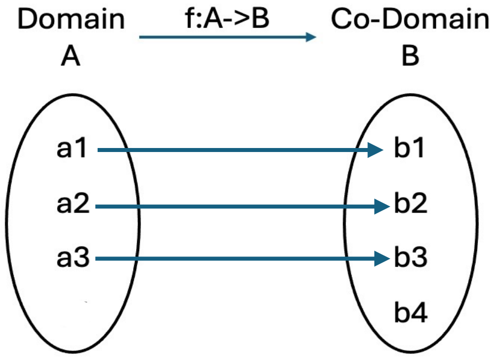
2. Surjective / Onto Mapping: here f(A) = B i.e., Range=Co-domain for f:A->B

3. Bijective / One-to-one Correspondent: A function $f:A \rightarrow B$ is bijective or one-to-one correspondent if and only if f is both injective and surjective

4. Non-Injective /Many-to-one Mapping: A function $f:A \rightarrow B$ is many-to-one if more than one elements in A have the same function-images in B

5. Into Mapping: A function $f:A \rightarrow B$ is into if there is at least one element in B which is not the f-image of any element in A. i.e., $f(A) \subset B$

6. Non-Surjective / One-to-Many Mapping: A single element in the domain maps to multiple elements in the codomain.



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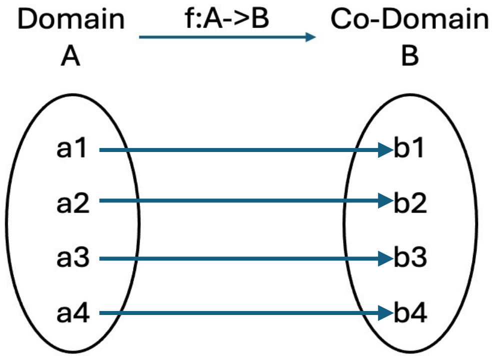

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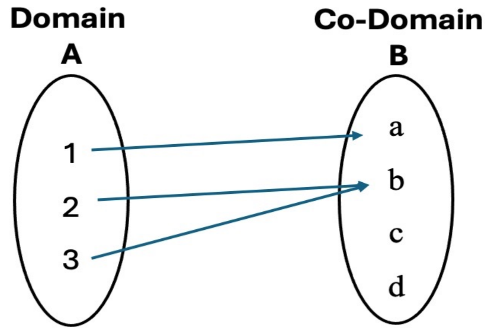

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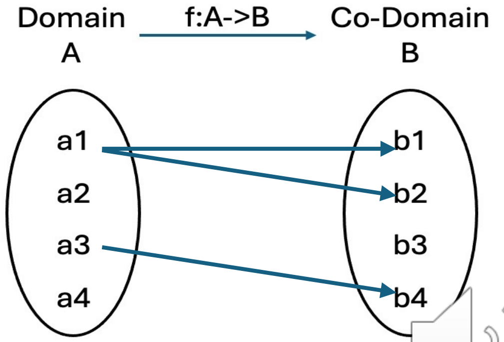

6

2. Sets and Relations

1. Set Operations
2. Representation and Properties of Relations
3. Equivalence Relations
4. Partially Ordering.

1. Set
2. Cardinality of a Set
3. Representation of a Set: Roster/Tabular Form and Set Builder Notation
4. Types of Sets (12): Finite, Infinite, Empty/ Null/Void, Singleton/Unit, Universal, Power, Equal, Equivalent, Overlapping, Disjoint
5. Set Operations (6): Union:∪, Intersection:∩, Difference/Relative Complement A−B or $A\setminus B$, Symmetric Difference: AΔB, Complement, Cartesian Product/Cross Product, Partitioning
6. Bell Numbers
7. Relations
8. Types of relations(5): Binary, Empty/Void, Full/Universal, Inverse, Identity
9. Reflexive v/s Identity Relation
10. Properties of Relations(6): Reflexive, Irreflexive/Anti-Reflexive, Symmetric, Anti-Symmetric, Asymmetric, Transitive
11. Equivalence Relation
12. Partially Ordering
13. Number of possible relations on 2 sets:
14. Representation of Relations(4): Matrix, Directed Graph, Table, Mapping
15. Function
16. Inverse of a Function
17. Composition of Functions
18. Types of mapping(6): Injective/One-to-one, Surjective/Onto, Bijective/One-to-one Correspondent, Non-Injective/Many-to-one, Into, Non-Surjective/One-to-Many